Solvay Brussels School Economics & Management

ULB

Risk Management and Governance
Interest rate risk management

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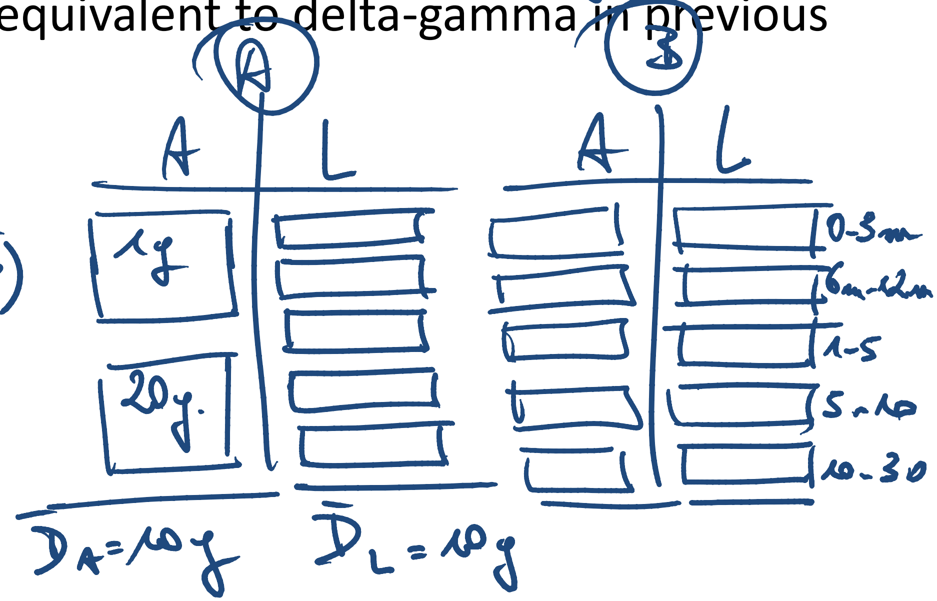
Introduction

- Interest rate risk (IR risk) is more difficult to manage
  - Various interest rates in each currency, not perfectly correlated
  - We need a function describing the variation of the rate with maturity, the *term structure of interest rates* or *yield curve*.
  - The points of the curve do not show only parallel movements.

value (rate)

reinvestment rate of coupons

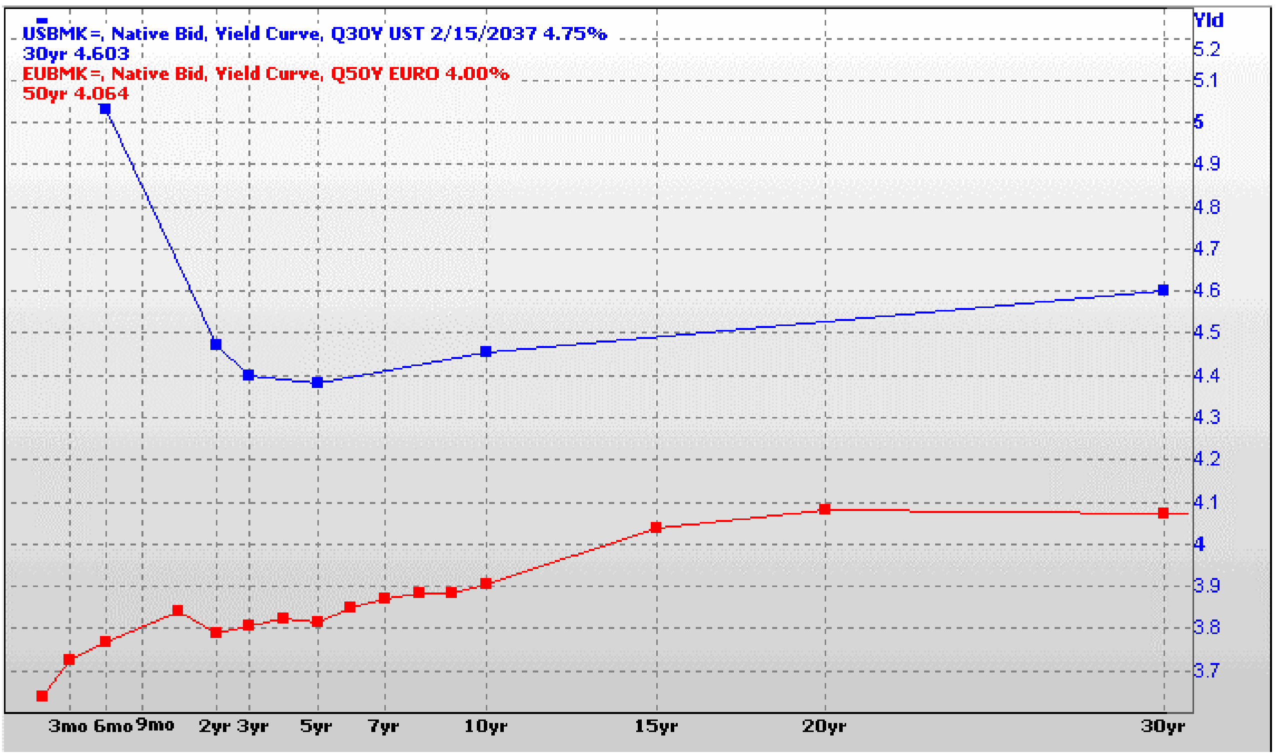
- Tools available
  - Duration and convexity measures (equivalent to delta-gamma in previous slides)
  - Partial durations
  - Multiple deltas
  - Principal component analysis PCA (ACP)
  - Evolution around the latter like ICA



Zero-rates

$$P_{zero\text{-}coupon} = \frac{100}{(1+r_{0,5})^5}$$

- *n*-year zero-coupon interest rate =
  - Interest earned on an investment starting today and lasting *n* years.
  - *n*-year spot rate, *n*-year zero rate, *n*-year zero $r(0,t)$ and $r^c(0,t)$
  - Annual compounded and continuous compounded rates:
- The term structure of zero-rates is also called the *zero-curve*.



Forward rates

- Annual compounded version

$$\left(1+r(0,t_1)\right)^{t_1}\left(1+f(t_1,t_2)\right)^{t_2-t_1}=\left(1+r(0,t_2)\right)^{t_2}$$

$$\rightarrow f(t_1,t_2)=\left[\frac{\left(1+r(0,t_2)\right)^{t_2}}{\left(1+r(0,t_1)\right)^{t_1}}\right]^{t_2-t_1}-1$$

- Continuously compounded version (or uncompounded with effective annual rates)

$$r(0,t_1)\times t_1+f(t_1,t_2)\times(t_2-t_1)=r(0,t_2)\times t_2$$

$$\rightarrow f(t_1,t_2)=\frac{r(0,t_2)\times t_2-r(0,t_1)\times t_1}{(t_2-t_1)}$$

- Practitioners use also the previous approach when they work with intra-annual forwards

Some forward rates

0#EURFSSM EUR FSSM MATRIX LINKED

	6M 05MAR07 REUTERS RTR	1Y 05MAR07 REUTERS RTR	18M 05MAR07 REUTERS RTR
1Y	4.052	4.007	3.998
2Y	4.025	4.002	4.005
3Y	4.021	4.011	4.017
4Y	4.026	4.026	4.041
5Y	4.043	4.048	4.065
6Y	4.062	4.072	4.094
7Y	4.087	4.102	4.123
8Y	4.113	4.129	4.153
9Y	4.139	4.157	4.181
10Y	4.166	4.183	4.207
12Y	4.211	4.228	4.249
15Y	4.264	4.279	4.299
20Y	4.310	4.320	4.333
30Y	4.295	4.300	4.306
40Y	4.257	4.259	4.263

Empirical research and expectations

- The unbiased expectations hypothesis (UEH)
  - "The forward rate is an unbiased predictor of the future spot rate"
  - It cannot be true theoretically and almost surely cannot be true in reality
  - The UEH implies that, e.g. for a zero-coupon bond... $f_0(T) = \mathrm{E}\left[S(T)\right]$
  - By definition $S(0) = \mathrm{E}\left[S(T)\right] - S(0)\left(e^r - 1\right) - \pi$
  - To prevent arbitrage, we know that $f_0(T) = S(0)e^r$
  - Using the last two equations, we get $f_0(T) = \mathrm{E}\left[S(T)\right] - \pi$
- The local expectations hypothesis (LEH) – equivalent to AOA
  - With AOA $f_0(T) = \mathrm{E}^Q\left[S(T)\right]$
  - The equivalence of the forward price and expected spot price is, however, true only for one-period-ahead forward prices.
  - The expected returns, taken using the martingale probabilities, of any strategies involving any bonds of any maturity, are equivalent and equal to the one-period spot rate, i.e., the shortest interest rate in the market
- The market segmentation hypothesis
- The liquidity premium hypothesis

Ref: Don Chance

LIBOR rates...

- LIBOR
  - *London Interbank Offered Rate*: 1m, 3m, 6m and 12m rates for wholesale deposits from banks within another one.
  - The receiving bank must have an AA rating.
  - The committee of banks fixing the LIBOR has a guaranteed AA rating.
  - Short-term rate used for the floating leg of the swap.
- LIBID

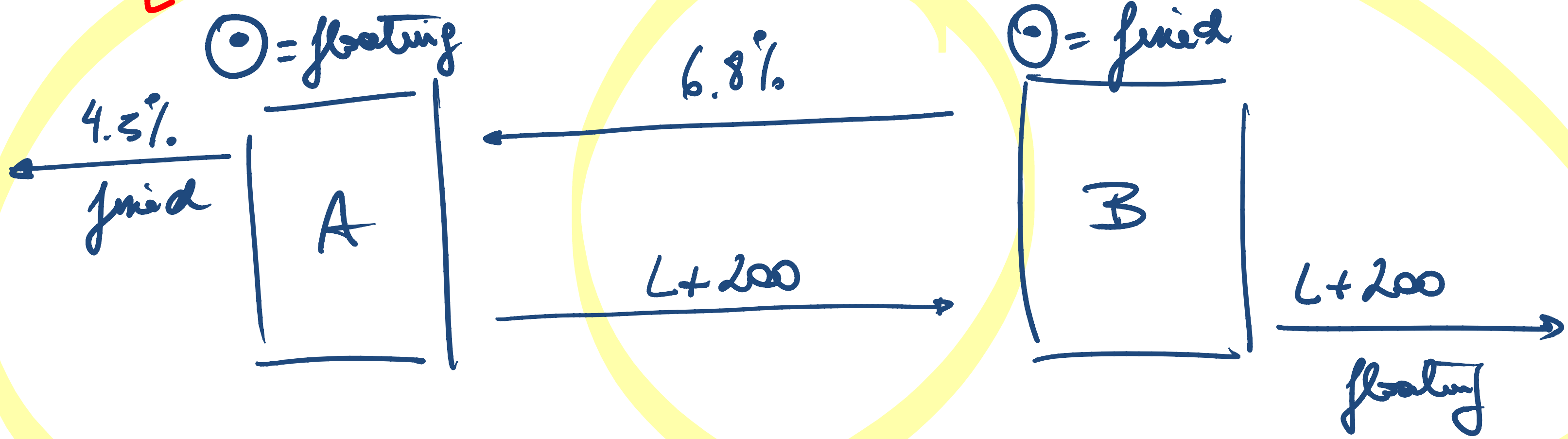
EURIBOR
EONIA
EIB rates

Swap rates...

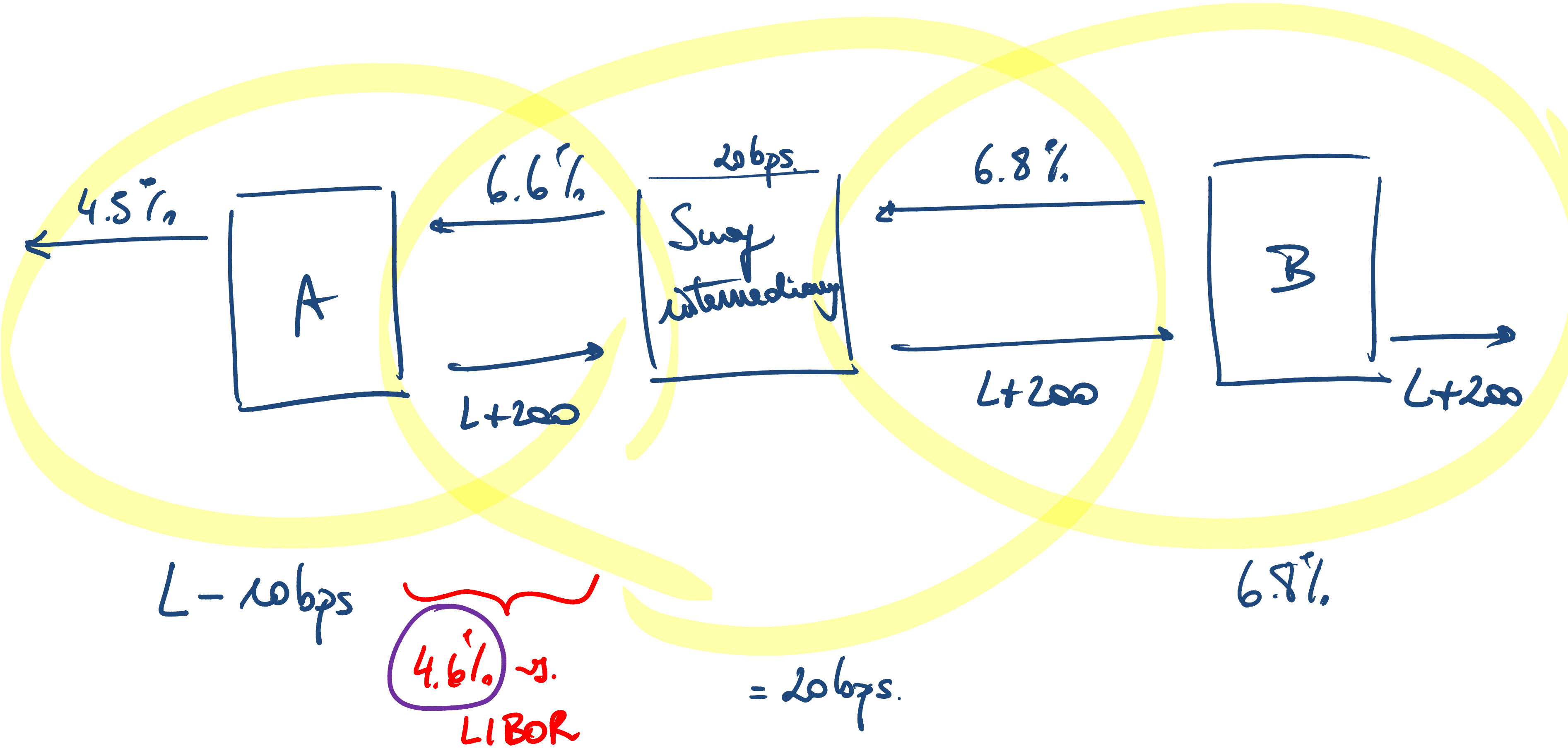
L+2% - 2.3% <=> L+10bps

$\underbrace{\text{L+2\% - 2.3\%}}_{\text{L - 30bps}}$

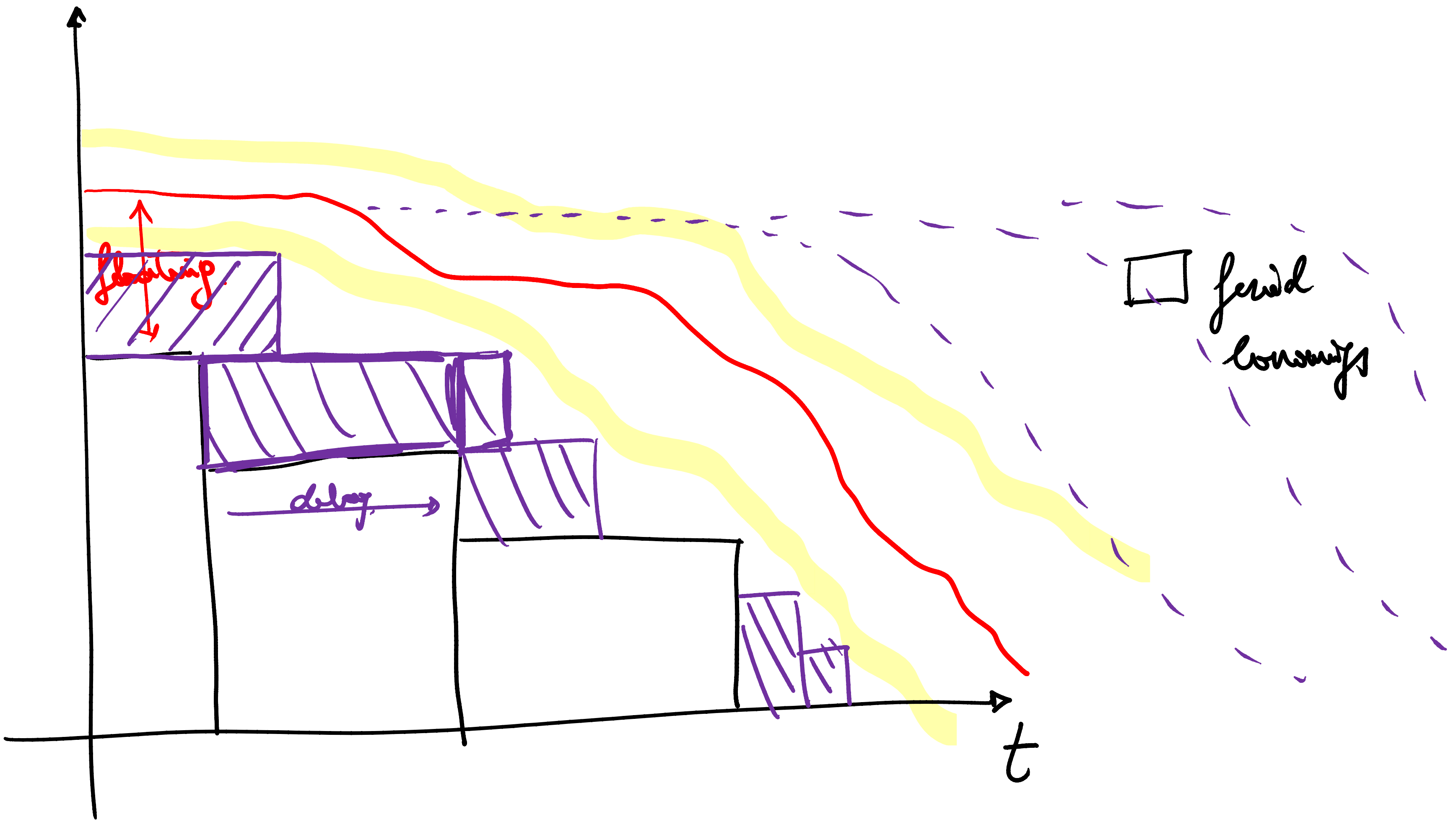
// 6.8%



		floating	fixed
A	AAA	LIBOR + 10 bps	4.5%
B	BBB	LIBOR + 200 bps	7%
		190 bps	2.5% = 250 bps


4.5%
A
6.6%
L+200
20bps.
Swap intermediary
6.8%
B
L+200
L+200
L-10bps
4.6%
LIBOR
= 20bps.
6.8%

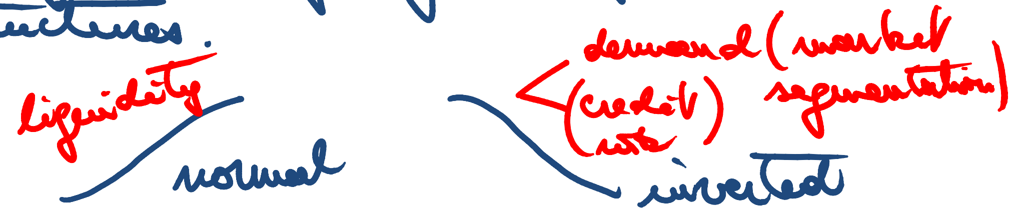
Dashboarding → Gap Analysis



Orange County's Yield Curve Plays

- What's a yield curve play?

"riding the curve" ⟹ taking advantage of non-flat structures.



- At Orange County
  - Robert Citron was very successful in 1992 and 1993 with these yield curve plays
  - In 1994, he decided to use *inverse floaters*: interest = fixed rate – floating rate and leveraging that position by borrowing at the short-term rate.
  - When short-term rate rised, the portfolio had lost \$1.5 billion and OC filed for bankruptcy.
- Same game played by the Savings and Loans in the 80's.

Bond pricing and Yields

- Bond prices

$$B(0)=\sum_{i=1}^{n} c_i\, e^{-r^c(0,t_i)\ \times{`}t_i} \qquad \text{or} \qquad B(0)=\sum_{i=1}^{n} \frac{c_i}{\left(1+r(0,t_i)\right)^{t_i}}$$

- Bond yield

yield-to-maturity

y such that

$$B^m(0)=\sum_{i=1}^{n} c_i\, e^{-y^c \times t_i} \qquad \text{or} \qquad B^m(0)=\sum_{i=1}^{n} \frac{c_i}{(1+y)^{t_i}}$$

where

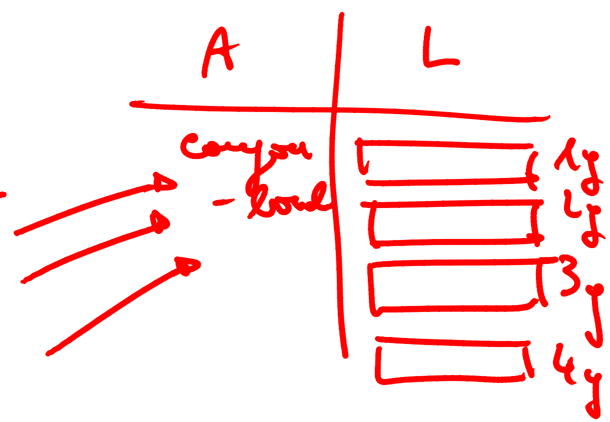
$B^m =$ market price of the zero-coupon bond

How do you determine government zero rates?

1. Looking at strips
   - Separate Trading of Registered Interest and Principal of Securities (STRIPS)
   - Prices of zero-coupons → perfect discount factors
   - Example: Suppose you observe the following prices

Maturity	Price for \$100 face value
1	98.03
2	94.65
3	90.44
4	86.48
5	80.00

$$\left(\frac{100}{90.44}\right)^{1/3} - 1$$



2. Using the bootstrap method
   - Take bond prices for the various maturities (must be a continuous set)
   - Extract the 1y discount rate for the 1y bond
   - Use that rate and extract the 2y discount rate for the 2y bond
   - And so on...

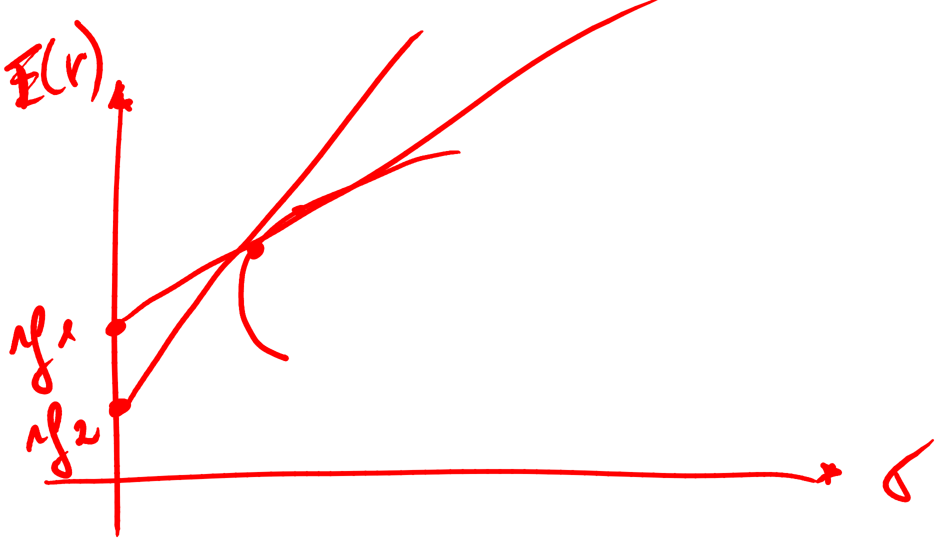
How do you determine government zero rates? (2)

3. Using discount, LIBOR, forward and swap rates
   - Combine these rates to generate the underlying zero-coupon curve
   - Extending the LIBOR beyond one year
     a) Create a yield curve that represents the rates a which AA-rated companies can borrow for periods of time longer than one year.
     b) Create a yield curve to represent the future short-term borrowing rates for AA-rated companies.

     In practice, we do (b).
   - Example

The risk-free rate

- It is usual to assume that the LIBOR/swap yield curve provides the risk-free rate
  - » Treasury rates are too low
    - ✓ Must be purchased by a variety of institutions to fulfill regulatory requirements
    - ✓ The amount of capital to support an investment in T-Bills and Bonds is lower than for other similar investments
    - ✓ Favorable tax treatment in the US



Macaulays' duration

- Duration
  - » For a zero-coupon bond: maturity = duration
  - » Weighted average of the times when the payments are made = weighted average of the maturities of zero-coupon bonds

$$\boxed{D = \sum_{i=1}^{n} t_i \left[\frac{c_i e^{-y t_i}}{B} \right]}$$

- Bond pricing and duration (with continuous compounding)

$$\Delta B = \frac{dB}{dy} \Delta y \qquad \left\{ \begin{array}{l} \text{Since } \; B = \sum_{i=1}^{n} c_i \, e^{-y^c \times t_i} \quad , \\ \\ \Delta B = -\Delta y \sum_{i=1}^{n} c_i \, t_i \, e^{-y^c \times t_i} \end{array} \right.$$

$$\Delta B = -B D \Delta y \quad \rightarrow \quad \boxed{\frac{\Delta B}{B} = -D \Delta y}$$

Modified duration $= \dfrac{D}{(1+y)}$

- Bond pricing and duration
  - » when y is expressed with annual compounding

$$\Delta B = -\frac{BD\Delta y}{1+y}$$

  - » with a compounding of m times during the year

$$\Delta B = -\frac{BD\Delta y}{1+y/m}$$

  - » If : $D^* = \dfrac{D}{1+y/m}$
  - » Then, we recover the following formulation

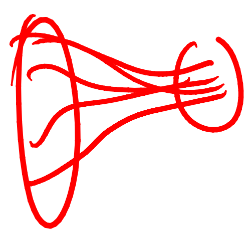
$$\Delta B = -BD^*\Delta y \quad \rightarrow \quad \boxed{\frac{\Delta B}{B} = -D^*\Delta y}$$

(!) D is (good) for:

① flat structures of interest rates

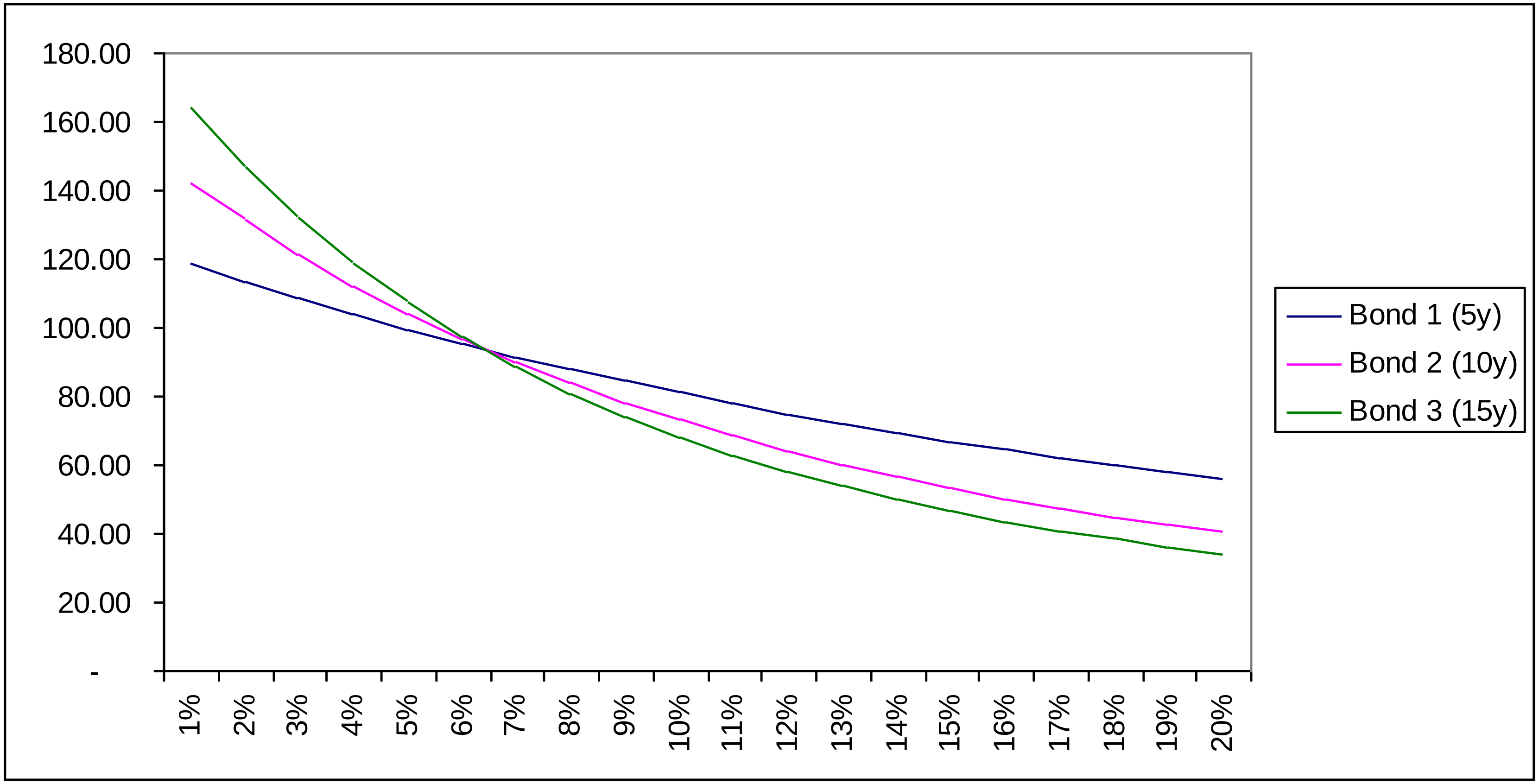
OR

② parallel changes of the term structure



Convexity

- Graph



Convexity

- Formulation (continuous-time)

$$C = \frac{1}{B}\frac{\partial^2 B}{\partial y^2} = \frac{\sum_{i=1}^{n} c_i t_i^2 e^{-y t_i}}{B}$$

$$\begin{cases} \text{Taylor: } \Delta B = \dfrac{\partial B}{\partial y}\Delta y + \dfrac{1}{2}\dfrac{\partial^2 B}{\partial y^2}(\Delta y)^2 \\ \text{Thus: } \dfrac{\Delta B}{B} = -D\Delta y + \dfrac{1}{2}C(\Delta y)^2 \end{cases}$$

- Formulation (discrete-time)

$$C = \frac{1}{B}\frac{\partial^2 B}{\partial y^2} = \frac{\sum_{i=1}^{n} c_i\, t_i\, t_{(i+1)} \Big/ (1+y)^{t_i}}{B}$$

Portfolio immunization

- For a portfolio P
  - Duration of the portfolio is the weighted average of the durations of the components

$$\text{Since } \Delta P = \sum_{i=1}^{n} \Delta X_i \qquad \text{we have that} \qquad D_{pf} = -\frac{1}{P}\sum_{i=1}^{n}\frac{\Delta X_i}{\Delta y}$$

  - The duration of the *i*th asset is

$$D_i = -\frac{1}{X_i}\frac{\Delta X_i}{\Delta y} \qquad \text{hence} \qquad \boxed{\underline{D_{pf}} = \sum_{i=1}^{n}\frac{X_i}{P}\underline{D_i}}$$

  - Idem for the convexity

- Therefore we have

$$\frac{\Delta P}{P} = -D_{pf}\Delta y + \frac{1}{2}C_{pf}(\Delta y)^2$$

Problems with duration?

big changes in Δr

non-parallel shifts.

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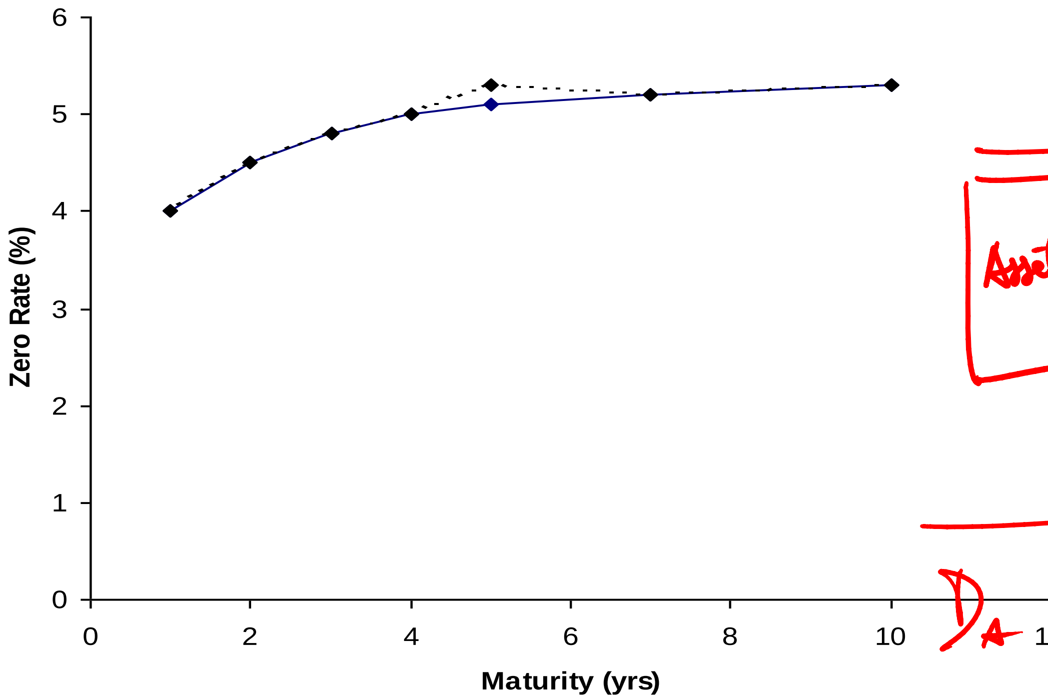
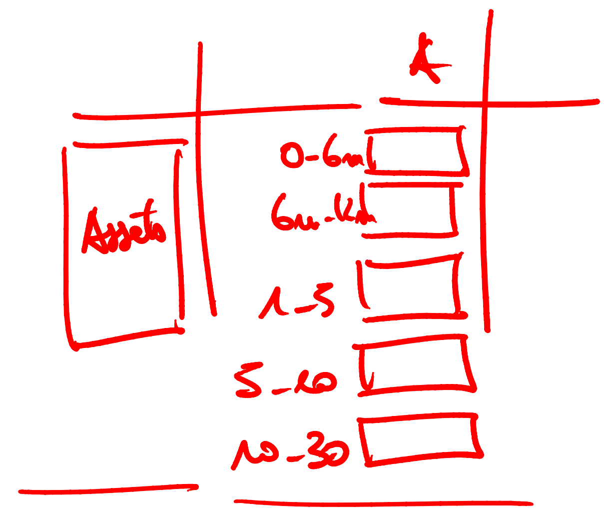
Calculating a partial duration

- Partial duration
  - » Formula $D_j^p = -\frac{1}{P}\frac{\Delta P_j}{\Delta r_j}$
  - » The sum of partial duration should equal the total duration

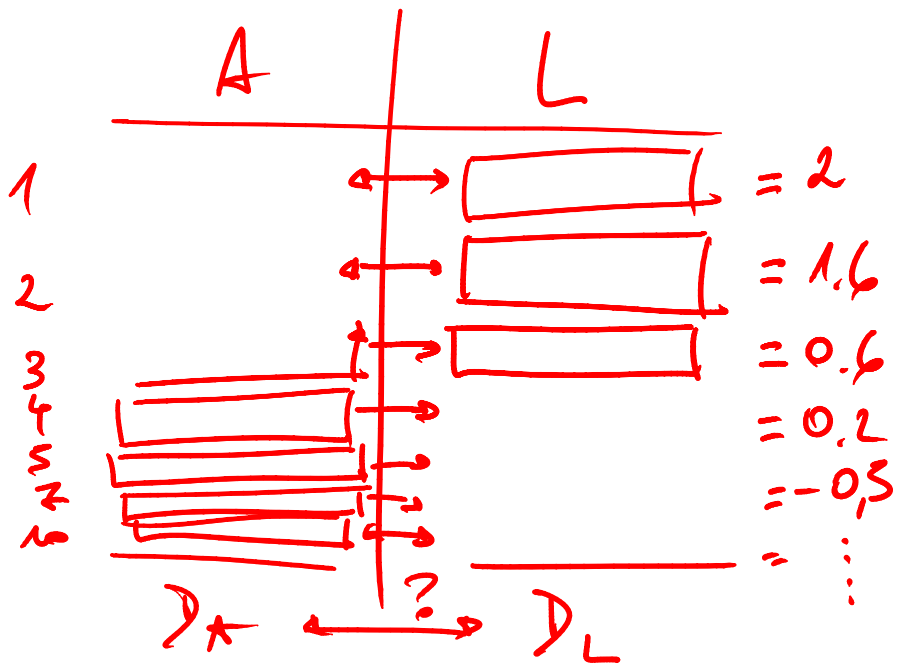




$D_A = D_A$

Example from Hull

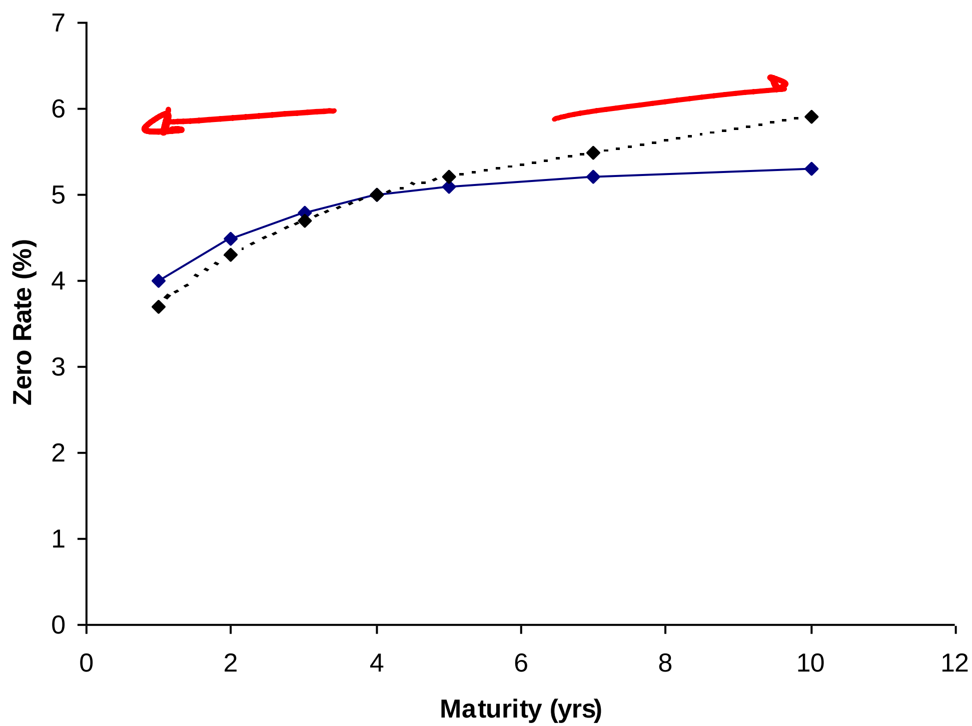
Maturity yrs	1	2	3	4	5	7	10	Total
Partial duration	2.0	1.6	0.6	0.2	−0.5	−1.8	−1.9	0.2



Combining partial durations

- Changes of -3*e*, -2*e*, -*e*, 0, *e* , 3*e*, 6*e* for a small *e* in the 1,2,3,4,5,7,10 y buckets

or any other combination for a given exercise.

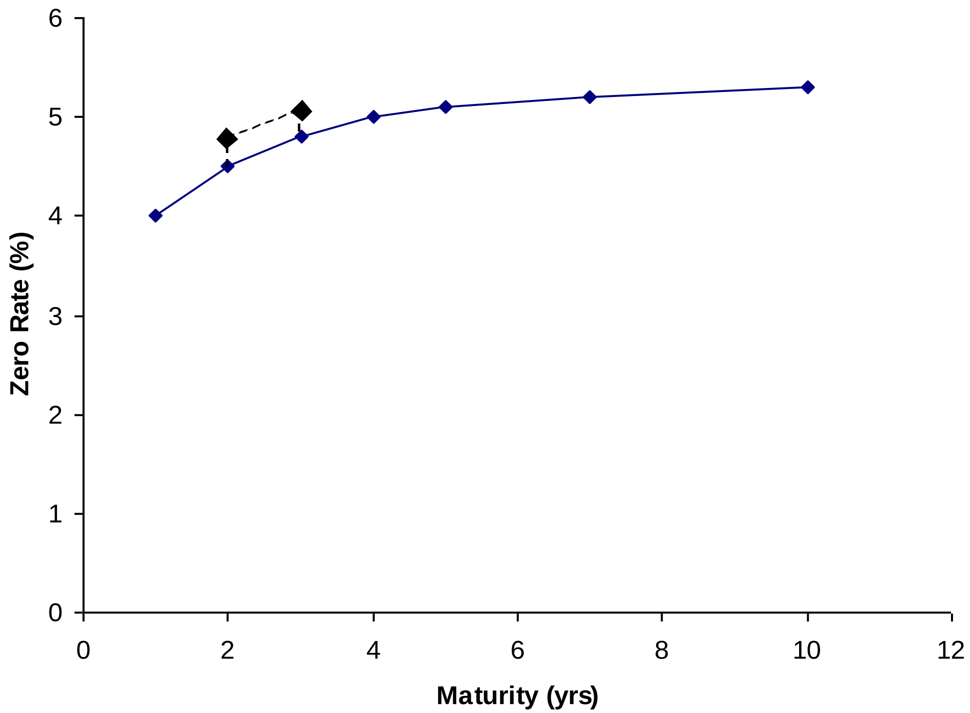


Interest rate deltas

- Definitions
  - » Change in value for 1bp parallel shift in the zero-curve
  - » Delta, DV01 or PVBP (Present Value of 1BP)
  - » Delta = Duration * Value of the portfolio * 0.0001
  - » As for the partial durations, it can be done for each point on the zero-coupon curve.
  - » The sum of the deltas should equal the delta of the portfolio

Change when one bucket is shifted

- Approach used in ALM
  - » GAP management
  - » Only one bucket is impacted by 1bp



Principal components analysis (PCA)

- The prior approach can lead to calculate 10 to 15 deltas for each curve
  - Quite overkill because variables are highly correlated between them
  - One idea would then to use historical data on movements in the rates and attempt to define a set of components that explain the movements.
- Idea
  - Attempts to identify standard shifts (or factors) for the yield curve so that most of the movements that are observed in practice are combinations of the standard shifts

$$y_t = b_1 \cdot X_{1_t} + b_2 \cdot X_{2_t} + \varepsilon_t$$

reduction of the information

$$y_t = b_1 X_1 + b_2 X_2 + \ldots$$

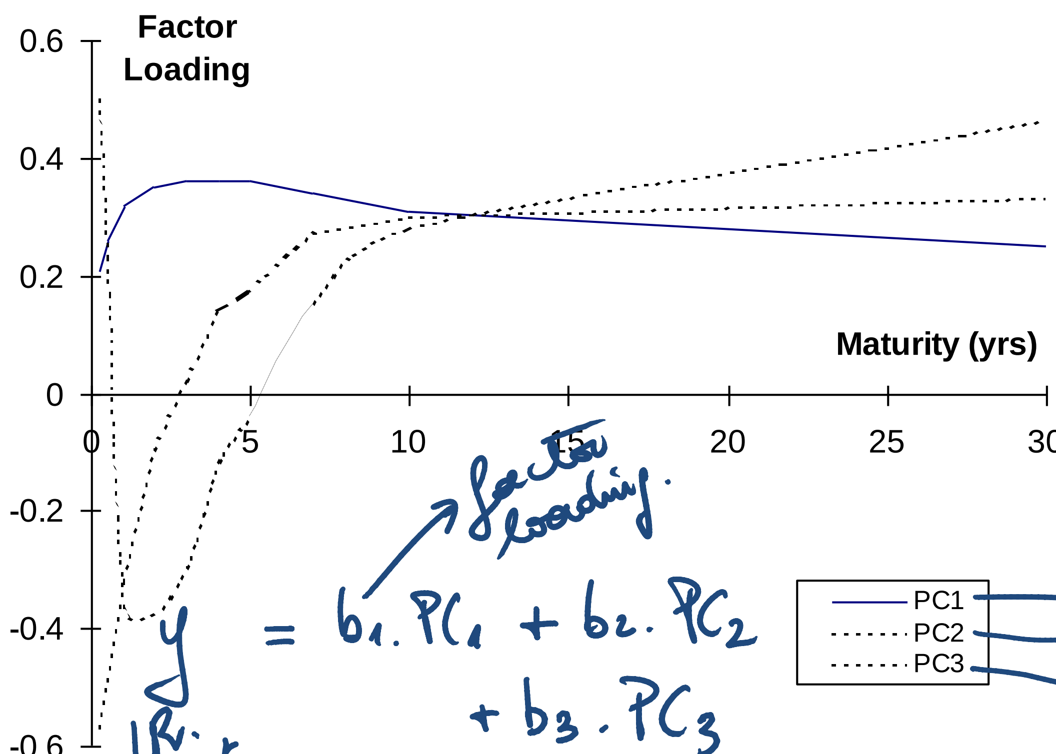
uses of PCA

+ # factors

Application to IRs

- Results of a study by Frye in 1997, published in *Risk publications*
  - The first factor is a roughly parallel shift (83.1% of variation explained)
  - The second factor is a twist (10% of variation explained)
  - The third factor is a bowing (2.8% of variation explained)

(the importance of a factor is measured by the standard deviation of its factor score)



factor loading.

$$\underbrace{y}_{IR_{i,t}} = b_1 . PC_1 + b_2 . PC_2 + b_3 . PC_3$$

PC1 → level
PC2 → slope
PC3 → bump / hump

Alternatives for Calculating Multiple Deltas

- Shift individual points on the yield curve by one basis point
- Shift segments of the yield curve by one basis point
- Shift quotes on instruments used to calculate the yield curve
- Calculate deltas with respect to the shifts given by a principal components analysis.

Gamma for Interest Rates

- Gamma has the form $\dfrac{\partial^2 \Pi}{\partial x_i \partial x_j}$

  where xi and xj are yield curve shifts considered for delta

- To avoid too many numbers being produced one possibility is consider only i = j
- Another is to consider only parallel shifts in the yield curve
- Another is to consider the first two or three types of shift given by a principal components analysis

Vega for Interest Rates

- One possibility is to make the same change to all interest rate implied volatilities. (However implied volatilities for long-dated options change by less than those for short-dated options.)
- Another is to do a principal components analysis on implied volatility changes.

References

- Books & Notes
  - RMH: Chap. 7
  - Don Chance teaching notes